15. Let m and b be real numbers and let the function f be defined by

$$f(x) = \begin{cases} 1 + 3bx + 2x^2 & \text{for } x \le 1 \\ mx + b & \text{for } x > 1 . \end{cases}$$

If f is both continuous and differentiable at $x = 1$, then

(A) $m = 1, b = 1$

(B) $m = 1, b = -1$

(C) $m = -1, b = 1$

(D) $m = -1, b = -1$

(E) none of the above

16. The function f is defined on all the reals such that $f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \le 1 \\ 3x + b & \text{for } x > 1. \end{cases}$

For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

(A) $k = -1, b = -3$

(B) $k = 1, b = 3$

(C) $k = 1, b = 4$

(D) $k = 1, b = -4$

(E) $k = -1, b = 6$

9. Let f be defined by $f(x) = \begin{cases} \dfrac{x^2 - 2x + 1}{x - 1} & \text{for } x \neq 1 \\ k & \text{for } x = 1. \end{cases}$

Determine the value of k for which f is continuous for all real x.

(A) 0

(B) 1

(C) 2

(D) 3

(E) none of the above

Ans

20. The function f is continuous at $x = 1$.

$$\text{If} \quad f(x) = \begin{cases} \dfrac{\sqrt{x+3} - \sqrt{3x+1}}{x-1} & \text{for } x \neq 1 \\ k & \text{for } x = 1 \end{cases}$$

then $k =$

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$ (E) none of the above

12. At $x = 3$, the function given by $f(x) = \begin{cases} x^2 & , x < 3 \\ 6x - 9, & x \geqq 3 \end{cases}$ is

$f(3) = 18 - 9 = 9$

$\lim_{x \to 3^+} = 9$

$\lim_{x \to 3^-} = 9$

(A) undefined

(B) continuous but not differentiable

(C) differentiable but not continuous

(D) neither continuous nor differentiable

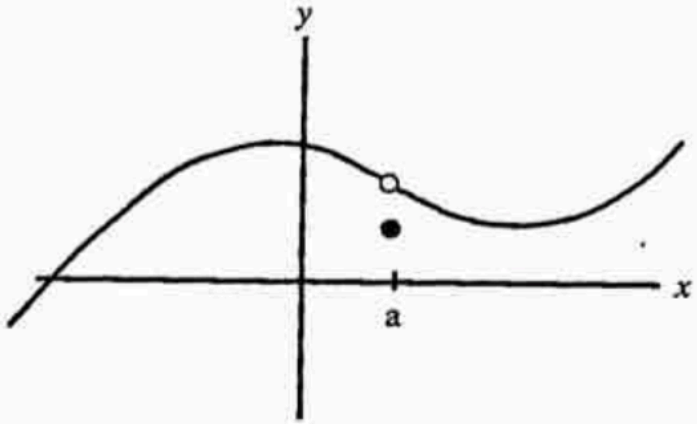
(E) both continuous and differentiable

2. If the function f is continuous for all positive real numbers and if $f(x) = \dfrac{\ln x^2 - x \ln x}{x - 2}$ when $x \neq 2$, then $f(2) =$

(A) -1 (B) -2 (C) $-e$ (D) $-\ln 2$ (E) undefined

Ans ______

1. The graph of a function f is shown to the right. Which of the following statements about f is false?



(A) f has a relative minimum at $x = a$.

(B) $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

(C) $\lim_{x \to a} f(x) \neq f(a)$

(D) $f(a) > 0$

(E) $f'(a) < 0$

Ans

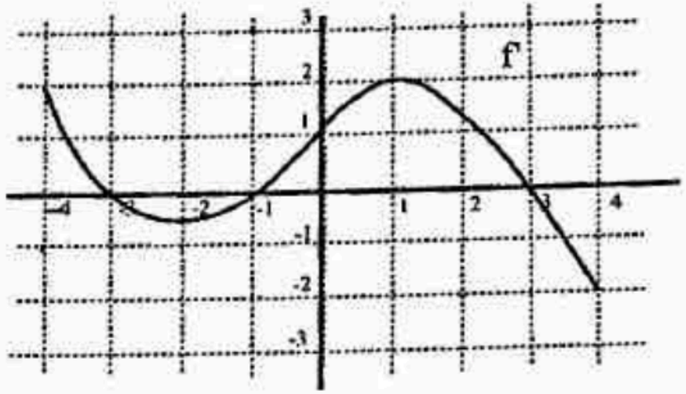
8. The graph of the **derivative** of a function f is shown to the right. If the graph of f' has horizontal tangents at $x = -2$ and 1, which of the following is true about the function f?

I. f is increasing on the interval $(-2, 1)$.

II. f is continuous at $x = 0$.

III. The graph of f has an inflection point at $x = -2$.

graph of the derivative of f



(A) I only (B) II only (C) III only (D) II and III only (E) I, II, III

4. Which of the following is true about the function f if $f(x) = \dfrac{(x-1)^2}{2x^2 - 5x + 3}$?

I. f is continuous at $x = 1$.

II. The graph of f has a vertical asymptote at $x = 1$.

III. The graph of f has a horizontal asymptote at $y = \frac{1}{2}$.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, III

25. Which of the following is continuous at $x = 1$?

I. $f(x) = |x - 1|$

II. $f(x) = e^{x-1}$

III. $f(x) = \ln(e^{x-1} - 1)$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, III

4. Which of the following functions is both continuous and differentiable at all x in the interval $-2 \le x \le 2$?

(A) $f(x) = |x^2 - 1|$

(B) $f(x) = \sqrt{x^2 - 1}$

(C) $f(x) = \sqrt{x^2 + 1}$

(D) $f(x) = \dfrac{1}{x^2 - 1}$

(E) none of these

4. The function f is continuous at the point $(c, f(c))$. Which of the following statements could be false?

(A) $\lim_{x \to c} f(x)$ exists

(B) $\lim_{x \to c} f(x) = f(c)$

(C) $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$

(D) f(c) is defined

(E) f'(c) exists

13. If $\lim_{x \to a} f(x) = L$, where L is a real number, which of the following must be true?

(A) $f'(a)$ exists.

(B) $f(x)$ is continuous at $x = a$.

(C) $f(x)$ is defined at $x = a$.

(D) $f(a) = L$

(E) None of the above